

About the helix structure of the Lund string

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ABSTRACT: The helix structure of the Lund string, first derived from studies devoted to the emission of soft gluons at the end of the parton cascade, may be at the origin of certain characteristic discrepancies observed in the low transverse momentum region at LEP and LHC. A study of the relation between different helix implementations and observable effects is presented. The model is extended to cover a multiparton string topology (result of parton shower), and compared with the experimental data. It is found that a helix-ordered string with a regular winding (proportional to the energy density stored in the string), is favoured by the inclusive single-particle spectra measured in the hadronic decay of Z^0 .

KEYWORDS: hadronization,string, screwiness.

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1. Introduction

The Lund fragmentation model [1, 4] uses the concept of string with uniform energy density to model the confining colour field between partons carrying complementary colour charge. The string is viewed as being composed of straight pieces stretched between individual partons according to the colour flow. The fragmentation of the string proceeds via the tunneling effect (creation of a pair of a quark and an antiquark from vacuum) with a probability given by the fragmentation function. The sequence of string break-ups defines the final set of hadrons, each built from a $q\bar{q}$ pair (ev.diquark in case of baryons) and a piece of string between the two neighbouring string break-ups. The longitudinal hadron momenta stem directly from the space-time difference between the break-ups.

The model gives a fair description of the available high-energy hadronic data and is therefore widely used in experimental particle physics. It reproduces particularly well the particle multiplicity and longitudinal profile (jet formation) but there are certain characteristic discrepancies between data and simulation which suggest the treatment of transverse momenta may not be entirely adequate (more in Section 6). It is therefore interesting to develop and study alternative models.

A very interesting work devoted to the study of the properties of the emission of soft gluons was published by Andersson et al. some time ago [2]. Under the assumption that the

generating current has a tendency to emit as many soft gluons as possible, and due to the constraint imposed on the emission angle by helicity conservation, it was shown that the optimal packing of emitted gluons in the phase space corresponds to a helix-like ordered gluon chain. Such a structure of the colour field cannot be expressed through gluonic excitations of the string and it needs to be implemented as an internal string property.

2. Fragmentation of the Lund string with helix structure

The implementation of a string with a helix structure radically changes the way hadrons acquire their transverse momentum. In the conventional Lund model [1], the transverse momentum of the hadron is the (vectorial) sum of the transverse momenta of the (di)quarks which were created via a tunneling process during the breakup of the string. The transverse momenta of newly created partons are randomly sampled from a gaussian distribution (with adjustable width) and their azimuthal direction is random.

In the helix ordered string, hadrons obtain their transverse momentum from the shape of the colour field itself, so that there is in principle no need to assign a momentum to new quarks in the string breakup. If we picture the colour field as a stream of soft gluons ordered at emission, we get the hadron transverse momentum by integration over the transverse momenta of soft gluons emitted in between the string break-up points which define the hadron, see Fig.1:

$$\vec{p}_t(hadron) = \int_{\Phi_i}^{\Phi_j} \vec{p}_t(gluon) d\Phi$$

where $\Phi_{i(j)}$ is the 'phase' of the helix (azimuthal angle) in the break-up point $i(j)$.

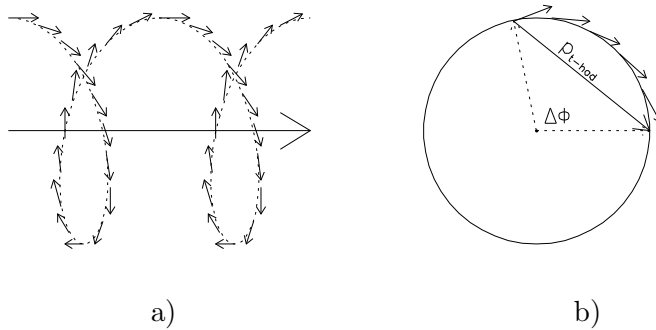


Figure 1: a) The helix structure of the string carried by colour connected chain of soft gluons. b) After fragmentation, the transverse momentum of a direct hadron is the integral of the transverse momenta of the soft gluons, integrated over the corresponding string piece.

The transverse momentum the hadron carries is thus entirely defined by the properties of the helix ordered field. This additional constraint translates into a loss of azimuthal degree of freedom in the string break-up, arguably the most significant consequence of the implementation of the helix string model in the fragmentation process. The underlying

helix structure is reflected in correlations between transverse and longitudinal components of hadrons which may lead to experimentally observable effects, depending on the actual form of the helix string.

So far, only one type of helix string parametrization was put under scrutiny [2, 3], and no convincing experimental evidence in favour of the helix string was obtained. The purpose of the present paper is to introduce an alternative helix string parametrization, to study and compare the observable effects, and to show that the helix string model (after modification) describes the hadronic data better than the standard Lund fragmentation model.

3. Parametrization of the helix string: theory

As a reminder, and for the sake of clarity, we reiterate the properties of the helix string introduced in [2]. Certain aspects of the original implementation which were not necessarily addressed in the original paper, but which are relevant for the discussion, will be pointed out.

3.1 The Lund helix model

In [2], the phase difference of the helix winding was related to the rapidity difference of the emitting current by the formulae:

$$\Delta\Phi = \frac{\Delta y}{\tau}, \quad (3.1)$$

where $\Delta\Phi$ is the difference in helix phase between two points along the string, Δy is their rapidity difference, and τ is a parameter. In the Lund model, the rapidity at a given point along the string is defined as

$$y = 0.5 \ln\left(\frac{k^+ p^+}{k^- p^-}\right), \quad (3.2)$$

where $p^{+,-}$ are the initial light-cone momenta of the endpoint quarks and $k^{+,-}$ their *fractions* defining a position along the string, see Fig.2.

The rapidity difference between two points along the string is then

$$\Delta y = 0.5 \ln\left(\frac{k_i^+ k_j^-}{k_i^- k_j^+}\right) \quad (3.3)$$

and it is related to the *angular* difference of points in the string diagram (Fig.2).

The evolution of the phase of the helix string defined according to (3.1) is illustrated for a simple $q\bar{q}$ string in the string diagram Fig.3. The phase is fixed by a random choice at one point of the diagram (in our example $\Phi=0$ at $[k^+, k^-]=[1,1]$) and is calculated for the rest of the diagram from Eq.3.1 with the help of Eq.3.2. The parameter τ is set to 0.3 for definiteness, its value is irrelevant for discussion of the qualitative features of the model. Please note that the density of helix winding increases with the distance from the string center, and becomes infinite near the turning points ($[k^+, k^-]=[0,1]/[1,0]$) where Eq. 3.3

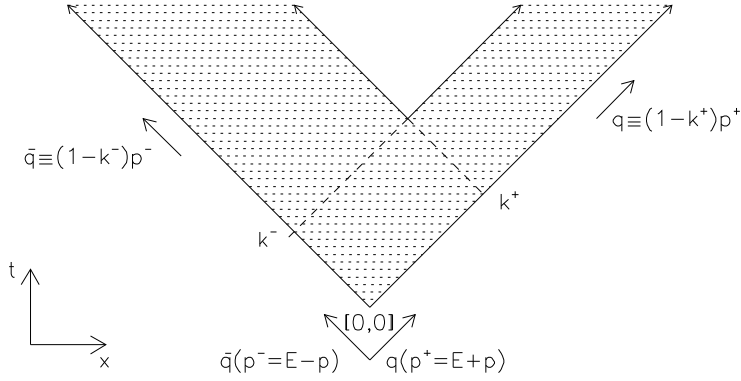


Figure 2: Evolution of the string in the rest frame of the $q\bar{q}$ pair, including the first string break-up. The partons lose their momentum as they separate and the string - the confining field - is created. The space-time coordinates can be obtained from the relation $[t,x]=(k^+p^++k^-p^-)/\kappa$ ($\kappa \sim 1 \text{ GeV/fm}$). The x direction is parallel to the string axis.

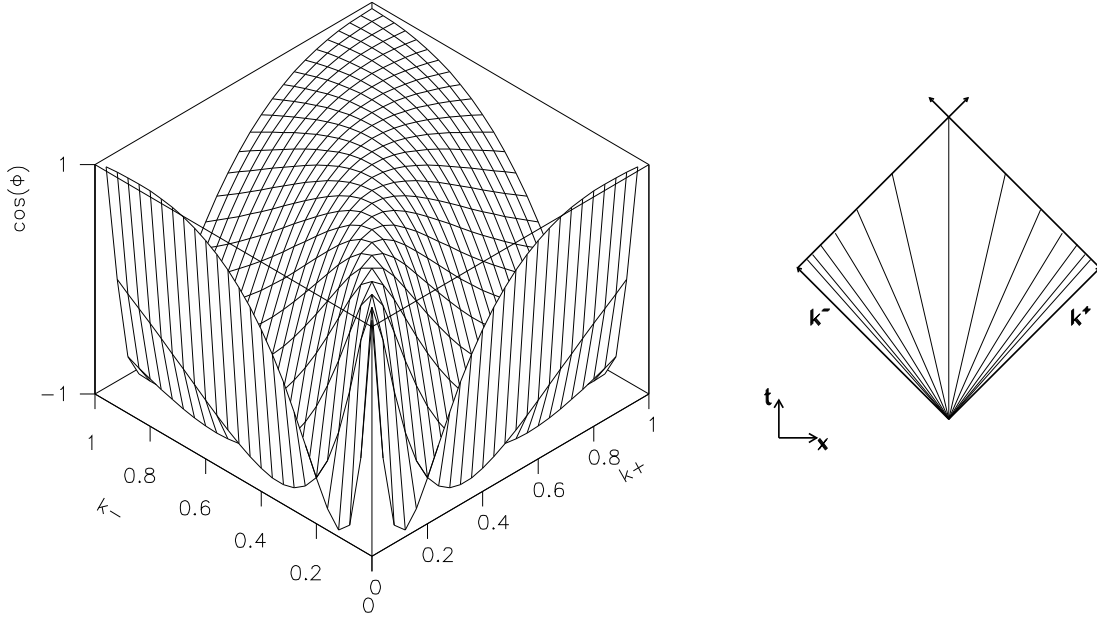


Figure 3: On the left: the phase of the helix winding along the string according to the Lund prescription (Eq.3.1) for parameter $\tau=0.3$. The phase was fixed by a random choice at one point. On the right: for better illustration of the phase evolution in the space-time coordinates, the points with equal phases are connected by lines in equidistant phase intervals. The singularity present in the model at the endpoint of the string is not graphically emphasized.

contains a singularity. This singularity does not affect the modelling of simple $q\bar{q}$ strings as long as the endpoint quarks do not acquire transverse momentum (a default solution in Pythia [4]) but the model needs some sort of regularization in case of multiparton string

configurations. (All studies in [2] were done using a simple $q\bar{q}$ string.)

3.2 The modified helix model

The presence of a singularity in the Lund helix model is one of the reasons why we wish to take a second look at the definition of the helix model. Also the requirement of the homogeneity of the string field which lies at the heart of the Lund fragmentation model seems to be poorly satisfied, given the difference of the helix winding at the middle of the string and near the string endpoints. It should be emphasized that we strictly adhere to the central idea of [2], namely the emergence of a helix-ordered gluon chain at the end of the parton cascade, and that we are merely looking into details of the helix parametrization.

We derive the alternative helix model studying the properties of an elementary dipole in the gluon chain on the basis of equation 3.3 from [2]. The squared mass of the dipole formed by colour connected gluons can be written as

$$s_{j,j+1} = k_T^2 2 [\cosh(\Delta y) - \cos(\Delta\Phi)] \quad (3.4)$$

where the transverse momenta of both gluons are set to k_T (for simplicity), Δy is the difference in rapidity, and $\Delta\Phi$ difference in azimuthal angle between gluons.

The original proposal for the helix string neglected the azimuthal difference in the search for gluon packing which would minimize the gluon distance yet satisfy the helicity conservation laws, and the distance between gluons was parametrized with the help of their rapidity difference. Here we intend to reverse the approach and develop a helix model which minimizes the rapidity difference between soft gluons and where the gluons are separated mainly in the transverse plane.

Under the assumption

$$\Delta\Phi \gg \Delta y \approx 0 \quad (3.5)$$

equation (3.4) reads

$$s_{j,j+1} = k_T^2 2 [1 - \cos(\Delta\Phi)] = 4 k_T^2 \sin^2\left(\frac{\Delta\Phi}{2}\right) \quad (3.6)$$

and the distance d between gluons, as introduced in [2], becomes

$$d_{j,j+1} = \sqrt{s_{j,j+1}/k_T^2} = 2 \left| \sin\left(\frac{\Delta\Phi}{2}\right) \right| \quad (3.7)$$

The condition $d \geq \frac{11}{6}$, derived from helicity conservation restrictions, is satisfied for $\Delta\Phi > 2.3$ rad. Since there is no constraint on the length of the gluon chain ordered in azimuthal angle, the number of soft gluons in the chain will depend on k_T and the energy available for string build-up. It has to be stressed however that we assume the emergence of the ordered helix field occurs in parallel with the 'homogenization' of the string field in which the interactions between field quanta redistribute the longitudinal momenta of field creating partons, and that we can describe the string with the help of uniform energy density and string tension, much as the standard Lund fragmentation model does. .

We set the difference in the helix phase to be proportional to the energy stored in between two points along the string

$$\Delta\Phi = \mathcal{S} (\Delta k^+ + \Delta k^-) M_0/2, \quad (3.8)$$

where M_0 stands for the invariant mass of the string, $\mathcal{S}[\text{rad/GeV}]$ is a parameter, and fractions $\Delta k^+ = |k_j^+ - k_{j+1}^+|$, $\Delta k^- = |k_j^- - k_{j+1}^-|$ define the size of the string piece.

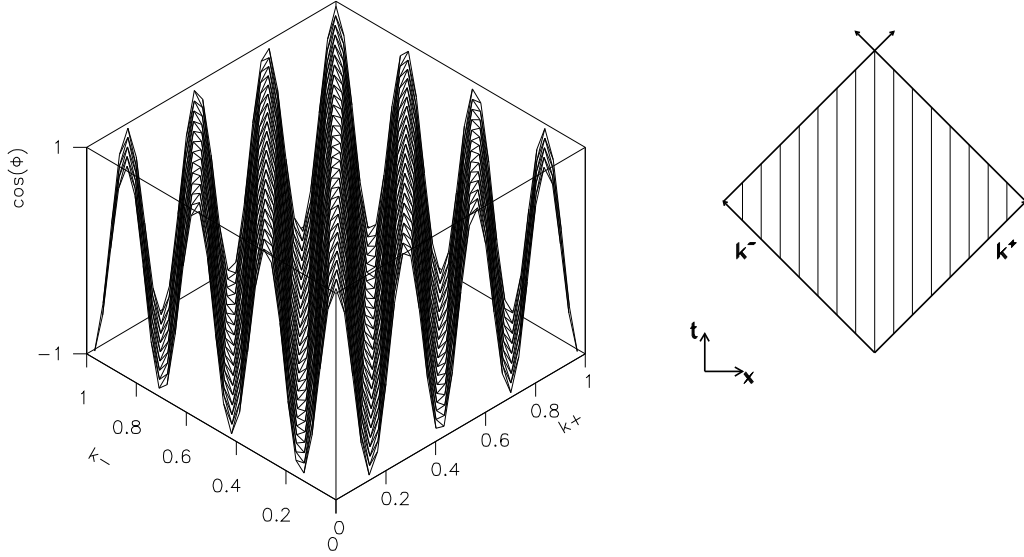


Figure 4: On the left: the phase of the modified helix (Eq.3.8) for parameter $\mathcal{S}=0.5$ rad/GeV and invariant mass of the string $M_0=91.22$ GeV. The phase was fixed by a random choice at one point. On the right: for better illustration of the phase evolution in the space-time coordinates, the points with equal phases are connected by lines in equidistant phase intervals.

As shown in the string diagram of Fig.4, the definition Eq.3.8 corresponds to a helix with a constant pitch along the string (proportional to the energy density of the string). The phase in the modified helix scenario is constant in time for a given point along the string axis, forming a stationary wave (similar to the interference pattern due to an emission from two sources).

4. Parametrization of the helix string: phenomenology

In this section we turn our attention to observable effects related to the helix ordered string. We shall first study a simple quark-antiquark system to get a better understanding of the differences between models.

In the Lund helix model [2], the phase difference is directly related to the rapidity difference. The fragmentation of the Lund string produces roughly one hadron per unit of rapidity. The hadrons, therefore, obtain – on average – a transverse momentum of about the same size, i.e. roughly independent of y , and the helix-like structure should be visible in their azimuthal angle difference as a function of the rapidity ordering.

The observable which should reveal such a behaviour was introduced in [2]

$$Screwiness(\omega) = \sum_e P_e \left| \sum_j \exp(i(\omega y_j - \Phi_j)) \right|^2, \quad (4.1)$$

where y_j, Φ_j stand for the rapidity and the azimuthal angle of final hadrons, P_e is a normalization factor and the parameter ω is the characteristic frequency of the helix rotation. The first sum goes over hadronic events, the second one over hadrons in a single event.

The expected signal for charged final particles is shown in Fig. 5. The presence of a Lund helix field is visible as a peak at $\omega \sim 1/\tau$, but the significance of the peak, with respect to standard Lund fragmentation, decreases with τ . There is some screwiness signal in the modified helix scenario, too, but it comes in a form of a multi-peak structure difficult to interpret. (It is worth remembering that the experimental study of screwiness [3] found a few percent difference between data and the standard Lund model but the signal did not exhibit a single peak shape.)

The observable effects stemming from a modified helix parametrization are however not restricted to the hadron ordering in the azimuthal angle (though we will come back to the question in section 7).

The modified helix introduces a strong correlation between the size of the fraction of string forming a hadron (i.e., the energy of the hadron in the string c.m.s.) and the size of

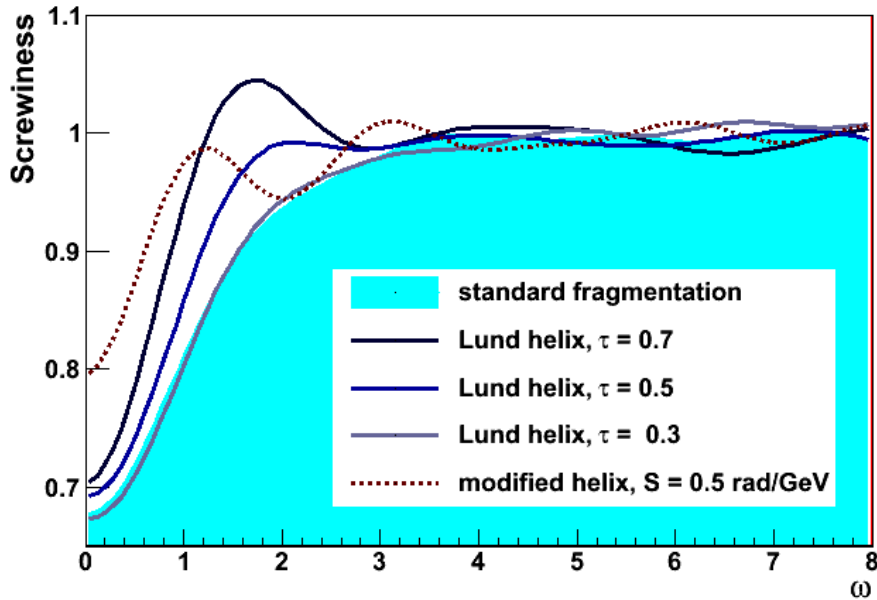


Figure 5: Screwiness signal obtained at the generator level in hadronic Z^0 decay (without parton shower), for various values of parameter τ in Lund helix model (3.1), and compared to standard Lund string fragmentation (histogram). A peak is expected at $\omega \sim 1/\tau$ but its significance is small for low τ values (< 0.3). Modified helix model (3.8) produces a multi-peak pattern (dashed line). Screwiness is calculated from final charged hadrons with $p > 0.15 \text{ GeV}/c$ and $|y| < 2$.

its transverse momentum. In the rest frame of the string,

$$p_T^2(\text{hadron}) = 4 r^2 \sin^2 \frac{\Delta\Phi}{2} = 4 r^2 \sin^2 \frac{S E_{had}}{2}. \quad (4.2)$$

where $r[\text{GeV}]$ (the 'radius' of the helix) is a parameter and $\Delta\Phi$ is the helix phase difference between the two break-ups which created the hadron.

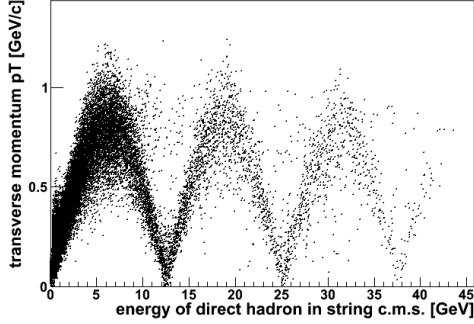


Figure 6: Correlations between the transverse momentum of a direct hadron and its energy in the string c.m.s. in the modified helix model, $r=(0.4\pm 0.1)\text{GeV}/c$, $S=0.5 \text{ rad/GeV}$.

The correlations are visible in Fig. 6, where a clear structure appears in the distribution of direct hadrons. Experimentally, we never observe such a clear picture of string fragmentation, because it is smeared by the parton shower and decays. Still, these correlations leave trace in the inclusive p_T spectra, as shown in Fig. 7. Due to the exponentially falling distribution of hadron energy in fragmentation which governs the size of transverse momentum through Eq. 4.2, the modified helix model creates more hadrons with very low p_t but less in the region $p_t \approx 0.4\text{GeV}/c$ where the peak of the uncorrelated, gaussian distribution lies.

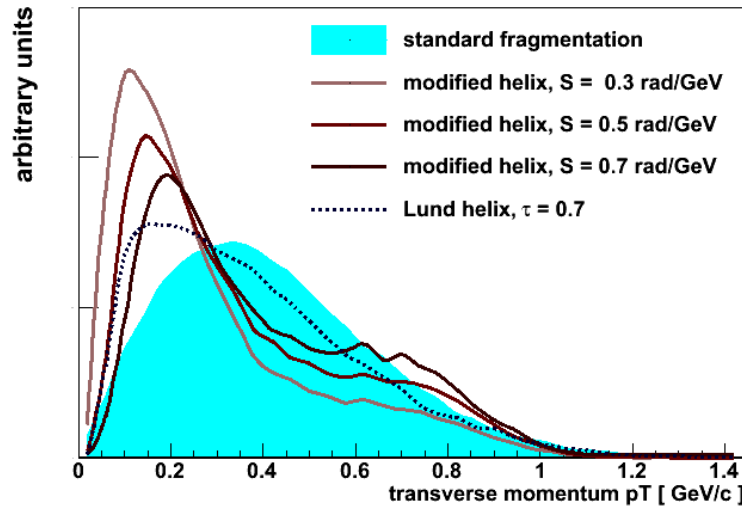


Figure 7: Inclusive transverse momentum distribution of direct charged pions, for the modified helix scenario (3.8) with different helix pitch, compared to the standard fragmentation. For comparison, the distribution obtained with the Lund helix model (3.1) is shown, too (dashed line).

For comparison, Fig. 7 also shows the spectrum obtained from the Lund helix model with $\tau = 0.7$, which exhibits qualitatively similar behaviour, albeit attenuated, as the modified helix model. (Generally speaking, we may expect observables designed for one helix scenario to show some effect in the other scenario, too, but weaker and somewhat distorted, as we saw in the case of the screwiness measure.)

The effect of the modified helix scenario on the inclusive p_T is strong enough to be readily visible in experimental data. As a matter of fact, a characteristic discrepancy in the low p_T distribution is visible both in LEP [5] and LHC [6] data, but before performing a direct comparison of data and the model, we need to make sure the model handles properly the multi-parton string we use for description of the real data.

5. Extension of helix model on multiparton string topology

The comparison of the helix string model with data requires the model to be extended to cover not only the simple case of $q\bar{q}$ system but also an arbitrary multiparton configuration corresponding to the emission of hard gluons from the quark-antiquark dipole. This is actually the most complicated part of the model implementation which requires some additional assumptions to be made.

The solution adopted for the modified helix scenario consists of two steps: first, the multiparton system is followed in space-time (every parton loses about 1 GeV of its energy per fermi in favour

of the developing string field, the energy loss of gluon is twice as much because the gluon participates in the creation of two adjacent string pieces) in order to find the way the string breaks into pieces, and to evaluate their respective masses (Fig.8). Every string piece is formed by a combination of the fractions of momenta of 2 partons, combinations and fractions depend on the distribution of partons in the phase space. The second step consists in calculation of the combined helix phase difference between endpoint quarks

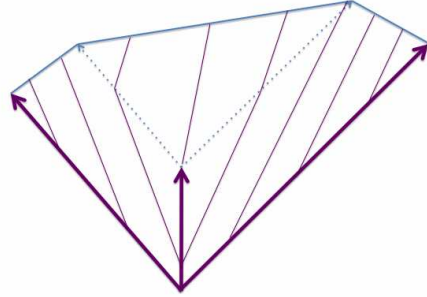


Figure 8: An illustration of helix phase evolution of the modified helix model in case of presence of a hard gluon kink on the string.

$$\Delta\Phi = S \sum_i M_i, \quad (5.1)$$

where the sum runs over all (ordered) string pieces and M_i is the mass of the i -th string piece. Since the phase of the modified helix string is constant at a given point along the string, it can be easily calculated from the relative distance from string endpoints. A convenient way of doing this is to use the energy fraction. For a given string break-up, for example, one can calculate the total energy of hadrons on the left and right side. The sum of hadron energies being identical to the sum of energies of ordered string pieces, it

is possible to associate the string break-up with a definite string piece, and to deduce the helix phase at a point of string break-up using Eq.3.8. The possibility to recover the helix phase anywhere in the string diagram in a simple way is due to the static nature of the helix field and cannot be applied to a non-static helix definition, in particular, to the Lund helix model.

It should be mentioned that the modified helix model extension for hard gluon kinks uses the assumption that the helix phase runs smoothly over the gluon kink, i.e. the helix phases at the connecting ends of adjacent string pieces coincide.

As much as we would like to perform a similar extension for hard gluon kinks using the Lund helix definition, we must admit the task goes beyond the scope of this paper. To start with, the Lund helix model corresponds to non-static helix form which evolves along the string with time. The evolution of the helix phase for a complex string topology was not addressed by the authors of the model in [2]. Second, there is the double discontinuity in the definition of helix winding at the gluon kink which needs to be stabilized somehow, and such a decision clearly belongs to the authors of the model.

Further details about implementation of the modified helix string scenario in Pythia code are given in Appendix A.

6. Model tuning and comparison with data

The helix string model (variant 3.8) has been tuned [7] to the DELPHI data [5] using a set of 6 simultaneously optimized parameters: helix radius r and pitch S , Lund fragmentation parameters a and b , effective coupling constant Λ_{QCD} and parton shower cut-off. It was known from previous studies that the helix string model significantly improves the description of p_T spectra, but the positive impact is actually much broader.

In Table 1 we see that the helix string model reduces the average χ^2/N_{bin} by more than one unit for the set of inclusive charged particle distributions and event shape variables used in the tune (619 data bins were used in total), for both types of parton shower used in the study (Pythia p_T ordered parton shower and Ariadne [9] parton shower). This is a remarkable result if one takes into account that the model actually removes a degree of freedom from the fragmentation process. While the performance of the (modified) helix string model in the description of data does not amount to a proof of the existence of the helix string structure in nature, it is a powerful indication that azimuthal ordering plays a significant role at the soft end of the parton cascade.

Data set	Pythia	helix + Pythia	Ariadne	helix + Ariadne
inclusive spectra + event shapes $N_{bin} = 619$	4075	2453	2453	1489

Table 1: The χ^2 difference between the DELPHI Z^0 data and models, summed over inclusive charged particle spectra and event shape variables [7] . The 'Pythia/Ariadne' labels distinguish between simulation setup using Pythia 6.421, resp. Ariadne 4.12 parton shower.

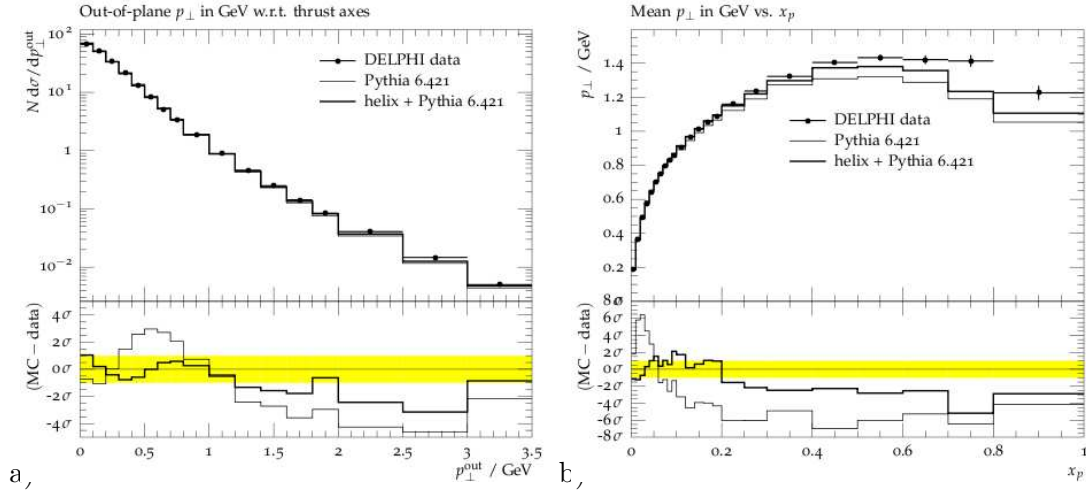


Figure 9: Comparison of inclusive charged particle distribution measured by the DELPHI Coll.[5] and Pythia 6.421 modelling: a) p_T projection on the Thrust-minor axis, b) average p_T as function of scaled momentum x_p .

The impact of the helix string model on the modelling is best seen in the comparison with measured charged p_T distributions. Figure 9 a) shows the projection of the transverse momentum of charged particles on the Thrust-minor axis (p_T^{out}). Data are compared with Professor tune of Pythia 6.421 [10] and with the tuned helix string model, implemented in a private version of Pythia 6.421 [11]. The characteristic discrepancy around $p_T \sim 0.5$ GeV is removed by the helix string model, and the smoother description of the low p_T region leads to a better adjustment of the parameters of the parton shower, as we can deduce from the improved description of the tail of the distribution.

Of special interest for our study is the dependence of the average p_T on the size of the particle momentum, which should be sensitive to the type of correlations pictured in Fig. 6. As shown in Figure 9 b), this distribution is much better described by the helix string model than by the standard string fragmentation. On the basis of this particular observation and of the global results of the helix model tuning, we can conclude that the helix string model is favoured by the data.

Tuned helix parameters suggest a helix radius ~ 0.4 GeV/c (rather well constrained) and a helix winding $S \sim 0.7$ rad/GeV (with large uncertainty). The variance of the helix radius was set to 0.1 GeV/c, for simplicity. The tune did not attempt to resolve a possible flavour dependence of the model.

7. Other observables

Further experimental input for the modified helix scenario (3.8) can be expected from a study of the ordering of hadrons in the azimuthal angle. Ideally, if we would be able to order hadrons along the string according to the fragmentation chain, the azimuthal opening angle for any given pair of direct hadrons should be correlated with the sum of energy of hadrons laying in between (the helix pitch is proportional to the string energy density).

In analogy with Eq.4.1, we define

$$S_E(\omega) = P \sum_{event} \left| \sum_j \exp(i(\omega \sum_{k=0}^j E_k - \Phi_j)) \right|^2, \quad (7.1)$$

where Φ_j stands for azimuthal angle of a hadron, ω is a parameter, P is a normalization factor. The outer sum runs over events, and the inner sum over the hadrons ordered in rapidity, resp. longitudinal momentum, in a given event. Such an ordering is only approximative, but generator level studies suggest the updated screwiness measure is sufficiently sensitive to provide a signal in the presence of a helix-like ordering. Under a rather stringent selection of event topologies (in order to suppress smearing due to the parton shower), the signal should be visible as a peak at $\omega \sim S$, the parameter describing the density of helix winding in Eq.3.8. In practice, the selection can be done by rejection of events where the maximal hadron p_T (with respect to the Thrust axis) exceeds 1 GeV/c.

Fig.10 shows the result of a generator level study of the Z^0 hadronic decay, where $S_E(\omega)$ is calculated using final charged particles only. The study was done on $\sim 50k$ events per sample, retained after the $p_T < 1$ GeV/c cut (original samples contained 500k events each). A few percent signal appears in the helix string model in comparison with the standard string fragmentation.

There is also a possibility that the helix string “memory” is partially conserved in the decay of short lived resonancies, which would lead to an enhanced signal, but no study has yet been done to estimate the possible effect.

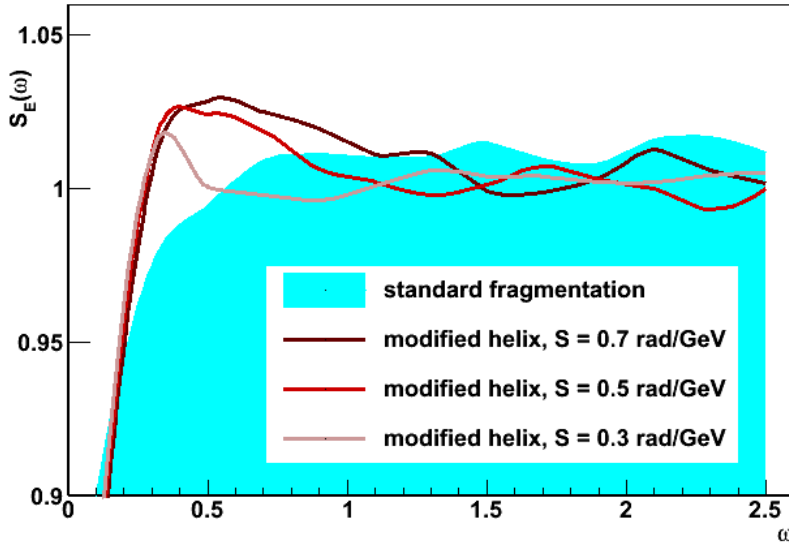


Figure 10: Updated screwiness measure (Eq.7.1) signal in the modified helix scenario. Generator level study with Pythia 6.421 using charged final particles from the hadronic Z^0 decay.

8. Conclusions

The idea of helix-like ordered gluon field definitely deserves a special attention. Further experimental evidence is needed to firmly establish the existence of such a phenomenon, but we are clearly touching a sensitive point in the modelling of soft QCD interactions. If we assume the helix ordering is regular, with pitch proportional to the distance along the string (for homogenous string field), the improvement of the description of the Z^0 data which can be achieved with such a model is significant.

For the moment, we don't have enough information to decide for, or against, a particular helix scenario, because the interplay between parton shower and formation of helix string needs further clarification. Hopefully, the indirect evidence gathered in this paper will contribute to a revival of interest for the model, in all existing variants.

Acknowledgments

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A. Pythia implementation of the modified helix scenario

The particularities of the helix string fragmentation have been discussed in section 6.2 of [2]. In the modified helix model, as in the original Lund helix proposal, the transverse momentum of the newly created hadron is determined by the sampling of the longitudinal momentum. The algorithm adopted in [11] approximates the fragmentation function (Eq.6.8 in [2]) by

$$f(z) = N \frac{(1-z)^a}{z} \exp\left(-\frac{b}{z}(m_h^2 + p_T^2(z))\right) \rightarrow N \frac{(1-z)^a}{z} \exp\left(-\frac{b}{z}(m_h^2 + r^2)\right) \quad (\text{A.1})$$

where r is the modified helix radius, which means hadrons are initially sampled as having the transverse momentum $|p_T| = r$. Once the hadron momentum is calculated in this approximation, its energy is preserved while the transverse and longitudinal components are adjusted to follow the shape of the helix field. In case there is no kinematical solution, the z fraction is resampled with the new p_T estimate.

The modified helix model is implemented in Pythia 6.421 [4] via private version of the fragmentation routine PYSTRF. The following Pythia parameters and switches are used for steering:

- MSTU(199)= 0/2 (standard fragmentation/modified helix fragmentation)
- PARJ(102)= * (helix radius r [GeV/c], replaces PARJ(21))
- PARJ(103)= * (helix radius variance [GeV/c])
- PARJ(104)= * (parameter S [rad/GeV])
- PARJ(109)= 0.001 (azimuthal angle tolerance in the iterative search of the string break-up solution conform to Lund fragmentation rules *and* helix string parametrization)

The modified helix fragmentation algorithm is about 10-20% slower than the standard Pythia fragmentation, with failure rate well below a per mille level.